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MATHEMATICAL BEAUTY MADE AUDIBLE: MUSICAL AESTHETICS IN PTOLEMY'S *HARMONICS*

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THE BEAUTIES OF MUSIC are celebrated in innumerable passages of ancient literature. But many Greek writers reserve their most ardent aesthetic admiration for items of an unexpected sort, not the compositions or performances they heard at symposia and religious ceremonies or in the theater, but the formal structures—such things as scales and patterns of attunement—on which melodies are built, and which provide them with their musical coherence. This perspective is particularly characteristic of authors who embrace the mathematical style of harmonic theory, in which musical intervals are expressed as ratios of numbers, and who hold that the principles to which musically acceptable structures and relations must conform are principles proper to mathematics. It is often labeled as the "Pythagorean" approach to the discipline, and it was indeed pioneered by Pythagoreans in the fifth century B.C.E.; but this label disguises the fact that from the fourth century onward it was adopted by the great majority of musical theorists and by philosophers of almost every persuasion.[1]

A fragment from one of the lost works of Aristotle provides a good example of these writers' expressions of admiration for such structures; it is Aristotle in a rather different vein from anything familiar to us in the surviving treatises:

> *Harmonia* is heavenly, and its nature is divine, beautiful and marvelous [οὐράνιος, θεῖος, καλός, δαιμόνιος]. It is fourfold in its natural power, and thus has two means, the arithmetic and the harmonic, and its parts and magnitudes and excesses [μέρη, μεγέθη, ὑπεροχαί] are displayed in conformity with number and with equality of measure [ἰσομετρία]; for melodies acquire their structure within two tetrachords.[2]

1. Among philosophers who engaged with musical theory, the most significant authors who rejected this approach are two fourth-century Peripatetics, Aristoxenus in *Harmonic Elements*, together with his later followers, and (for quite different reasons) Theophrastus, whose elaborate arguments on the subject are quoted by Porphyry (frag. 716 Fortenbaugh). Other Peripatetics, including Aristotle himself, invariably turn to the resources of mathematical harmonics when relevant issues arise, as do Plato and the later Platonists, the Stoics, and of course the Pythagoreans of Hellenistic and Roman times as well as the early pioneers.

2. [Plut.] *De mus*. 1139B, printed as *Eudemus* frag. 47 Rose, *De philosophia* frag. 25 Ross. (The translation is my own, as are all others in this paper unless otherwise noted.) I shall not try to explicate all the recondite details of this fragment here; for an attempt to unravel some of its complexities and those of the comments that follow in *De musica*, see Barker 2007, 329–38.

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It is abundantly clear that the object whose praises Aristotle is singing, and which he describes explicitly as beautiful, καλός, is not "music" in the usual sense of the word, but the skeletal framework that sets in place the fundamental elements of a musical scale, just the bare bones upon which the flesh and blood of a composition may be hung, but in whose absence nothing will be music. He names it as *harmonia*, and in the sequel it becomes clear that he is using this multifaceted expression in a way that was familiar in the fifth century but was later only occasionally revived, to refer to the relation of the octave. As in the best-known example of this usage, a fragment of Philolaus (DK 44B6), it is the octave conceived not merely as a relation between two notes, but as a complex system with a precisely articulated internal structure. The two means that Aristotle mentions—which had been defined in a musical context by Archytas (DK 47B2) and deployed by Plato in his musical analysis of the World-Soul in the *Timaeus* (35b–36b)—locate the inner boundaries of the tetrachords that occupy the octave's upper and lower regions. These are the two other foundational notes of the structure, whose relations to the octave's boundaries (unlike those of the remaining four notes of an eight-note scale) are never altered, no matter which variety of scale is in play. The arithmetic involved is very simple. When intervals are represented as ratios of numbers, the ratio of the octave is 2:1. If we represent this ratio as 12:6, the arithmetical mean between the terms is 9 and the harmonic mean is 8. Both 12:8 and 9:6 give us the ratio 3:2, the ratio of the perfect fifth, while 12:9 and 8:6 give us the ratio of the fourth, 4:3. Then if the greater number is assigned to the higher note (as it usually is), the arithmetic mean corresponds to a note lying at a perfect fourth from the top of the octave and a perfect fifth from the bottom, and the harmonic mean to a note at a fourth from the bottom and a fifth from the top, creating a symmetrical pattern of interlocking concords that bind together the highest and lowest notes of the octave. Thus the mathematical description of a *harmonia* becomes an intellectually lucid explication of the pattern of relations that presents itself to our hearing as musically fundamental. As Aristotle puts it in the *Posterior Analytics*, it is the role of mathematical harmonics, ἁρμονικὴ ἡ μαθηματική, to explain the phenomena that perception, or "harmonics based on hearing," ἁρμονικὴ ἡ κατὰ τὴν ἀκοήν, draws to our attention (78b–79a). At the level of structure, musical and mathematical perfection coincide, and it is mathematical analysis that will reveal the true nature of the relations that so entrance our ears.

Aesthetic excellence is often credited also to the elementary relations that are the building blocks of structures such as the one that Aristotle describes, that is, to the individual concords themselves. We are told in the *Timaeus*, for example, that these concords give pleasure, ἡδονή, to people who lack understanding and also a superior kind of delight, εὐφροσύνη, to those who understand well, "because of the μίμησις of the divine *harmonia* that comes into being in mortal movements" (*Ti.* 80b). This must mean that those capable of this enlightened form of experience will recognize in the concords they hear the relations that integrate the whole system of the universe, represented in the mathematical structure of the World-Soul; and though Plato does not

say explicitly that their εὐφροσύνη is a response to an encounter with something beautiful, it is impossible to doubt that the perfection in which they rejoice is καλός in the highest degree.[3]

According to Plato, then, the characteristic and aesthetically pleasing quality of an audible concord is due to its μίμησις of a "delightful" mathematical relation underlying the divine *harmonia*; and similarly, according to Aristotle, it is the beautiful formal relations revealed by mathematical analysis that account for the musical impressions registered by the ear. So far as the concords are concerned, two consequences apparently follow. First, all audible concords must share some perceptible and aesthetically agreeable attribute that distinguishes them from intervals of all other kinds, and in the same way there must be an attribute of a mathematical sort that is peculiar to the ratios of concords and that justifies the description of such ratios as beautiful. Secondly, if there is to be any substance in the notion of μίμησις, or in the explanations envisaged in the *Posterior Analytics*, these two attributes, together with the kinds of aesthetic excellence that each of them possesses, must stand in some intelligible relation to one another. How, then, do the theorists and philosophers represent the musical attributes appreciated by our senses and the mathematical attributes that delight our minds, and how do they elucidate the relation between them?

In what follows, I shall put the complexities of scales and attunements aside, to concentrate only on the most elementary kinds of case, where beauty is said to inhere in simple concords (in due course, we shall see how other legitimately musical but nonconcordant intervals can also be brought into the picture). One of the crucial questions about them is readily answered, since the many writers who describe the distinctive perceptible quality of a concord display an unusual level of agreement. From an aesthetic perspective, a concord is regularly defined as a relation between two notes such that when they are sounded together, they blend into one another so completely that neither is heard as a separate item; they merge into a unified whole. Pairs of notes forming discords, by contrast, fail to blend in this way; in our perceptual field, each note retains its own identity in disagreement with the other, and they refuse to coalesce into a coherent unity.[4]

3. The noun εὐφροσύνη occurs nowhere else in Plato except *Cra*. 419d, and the cognate verb εὐφραίνεσθαι is also rare in his writings. But its resonances may perhaps be judged from the distinction drawn by Prodicus at *Prt*. 337c (despite the elements of caricature in Plato's representation of him): εὐφραίνεσθαι arises when we are learning and using our intelligence, whereas ἥδεσθαι is a response to bodily pleasures such as eating. In the verb's one appearance in *Timaeus* (37c), it describes the god's reaction when he sees his divine construction beginning to live and to move. Compare also Diotima's statements at *Symp*. 206d, where it is directly linked to the experience of beauty: "What is ugly is out of tune [ἀνάρμοστον] with everything divine, and what is beautiful [καλόν] is in tune with it. That is why Beauty [Καλλονή] is the goddess of destiny who rules over childbirth. Hence when someone laboring to reproduce [τὸ κυοῦν, "that which is pregnant"] comes close to what is καλόν he becomes serene and relaxes in delight [εὐφραινόμενον] and gives birth."

4. See, e.g., Pl. *Ti*. 80a–b, [Eucl.] *Sect. can*. 149.18–20 Jan; Nicom. *Harm*. 12, 262.1–6 Jan; and more elaborately Porph. *Harm*. 35.26–36.3 Düring, quoting a Platonist named Aelianus (perhaps the rhetorician and philosopher Claudius Aelianus). Similar accounts are given by many other writers from the fourth century B.C.E. onward.

The concords, then, in their audible guise, are objects of aesthetic admiration because they are paradigmatic examples of the harmonious integration of diverse elements, submerging their differences in a cooperative union. The mathematical counterpart of this aesthetically satisfying fusion must therefore be some comparable kind of agreement between the terms of the ratio of any concord; the terms cohere with one another in a way in which those of the ratio of a discord do not. So what is the mathematical feature of this relation that gives it this unifying power? The question is one that Plato asks explicitly,[5] and it demands an answer, but only one of the texts surviving from the period before Ptolemy shows any sign of addressing it directly. Other writers were consciously committed to the view that there is something peculiarly admirable about the ratios of the concords, but they make no attempt to account for it or to say what it is. Their privileged status is simply taken for granted, as for instance by Aristotle, who speaks of the terms of the ratios of concords as the εὐλόγιστοι ἀριθμοί, the "numbers with good ratios," without further explanation (*Sens.* 439b).[6]

The one passage in the earlier texts that engages directly with the issue appears in the *Sectio canonis* conventionally attributed to Euclid,[7] though the conclusion it reaches had probably already been articulated by Archytas or even by one of his predecessors. The writer first distinguishes three classes of ratio, the multiples (in which the greater term is a multiple of the smaller), the epimorics (in which the greater is equal to the smaller plus one simple part or unit-fraction of the smaller), and the epimerics (which in this text and many others include all the ratios that are neither epimoric nor multiple). He then notes that the terms of any multiple and epimoric ratio "are spoken of in relation to one another by a single name," and that the notes of a concord, unlike those of a discord, blend together in the way I have described. He concludes that it is therefore "reasonable" (εἰκός) to suppose that the terms of their ratios are numbers "spoken of in relation to one another by a single name," and that the ratios must therefore be multiple or epimoric (*Sect. can.* 149.11–14 Jan).

This conclusion was embraced by a good many later writers, despite its problematic consequence that one of the intervals regularly treated as a concord by writers outside the mathematical tradition can be no such thing.[8]

5. *Resp.* 531c, where Socrates sets the trainee philosophers the task of "investigating which numbers [not "which notes"] are concordant with one another and which are not, and in each case why." What concerns us here is the question "Why?"; Socrates makes no attempt to answer it himself.

6. Aristotle is discussing the ways in which black and white can be mixed to produce other colors. The context shows clearly that he takes the superior quality of the most attractive colors to be due to the superiority of the ratios between the components of the mixture; and he identifies these ratios explicitly with those of the concords. The two kinds of case are precisely parallel, and raise the same question about the ratios' attributes. But Aristotle does not raise the question, and nothing in his text suggests an answer.

7. The attribution appears in Porphyry and in some of the MSS, but most modern scholars reject it. I agree with this judgment, but unlike some others I take the work to have been written, more or less in the form printed in Jan 1895, at a date not far from Euclid's, around 300 B.C.E. For discussion of various competing views on the issues, see Barbera 1991, 3–36; Barker 2007, 364–70 (and with additional details, 370–410).

8. It is the octave plus a perfect fourth, whose ratio is 8:3, neither epimoric nor multiple. Like several later texts in mathematical harmonics, *Sect. can.* deals with this problem by passing it over in silence.

But it is far from clear that the reasoning by which the writer justifies it, and links it with the phenomenon of "blending," is adequate for the purpose. It has been very variously interpreted, but if it is given what I take to be its obvious sense, it appeals to nothing more than a feature of fourth-century mathematical language, in which any multiple or epimoric ratio could be designated by a one-word name (e.g., τριπλάσιος for 3:1, ἐπίτριτος for 4:3), whereas epimeric ratios could not.[9] I shall not pause here to defend this interpretation. If it is correct, however, the linguistic peculiarity to which the writer refers may indeed reflect an awareness that the terms of these ratios, and of no others, are so closely interconnected that they come together to form a unified whole. But by itself it reveals nothing about the nature of the mathematical features by which they are integrated, and which allow the notes corresponding to the terms to blend into a single sound. In any case, no matter how the writer's reasoning is understood, the principle itself states at best only a necessary and not a sufficient condition for the concordance of an interval, since most epimoric and multiple ratios are ratios of discords.[10] Hence the principle does not identify a feature that concords alone possess.

The conclusion drawn in the *Sectio canonis*, though not its reasoning, plays an important role in Ptolemy's discussion of the concords; and the earlier texts provide him also with another, even more fundamental, element in his argument. It is, in fact, the concept that I think is the main key to his understanding of music's mathematical structure and of musical beauty too, both the beauty that appeals to the mind and that which entrances the ear. The name that he gives to this concept is συμμετρία. Now appeals to συμμετρία are very familiar, of course, in Greek philosophical discussions of beauty. Plotinus goes so far as to assert that virtually everyone before him had defined beauty as συμμετρία; and though he rejects the definition he readily agrees that συμμετρία is one of the most significant guises in which τὸ καλόν manifests itself (*Enn*. 1.6). It is a central component of the accounts of perceptible beauty which appear in the *Kanôn* of Polyclitus, in Plato, in Chrysippus, and many others; it would be tedious and pointless to continue the catalogue. But when one comes to examine how these authors use the term, it turns out in most cases to be disconcertingly vague. One thing that it almost never designates is the notion suggested by a direct transliteration, "symmetry." "Balance" and "due proportion" are nearer the mark, but we rarely find any precise specification of the proportions that are to count as σύμμετρα, still less any explanation of why those proportions and no others are the right ones.

The absence of a clear definition of συμμετρία in such contexts is particularly puzzling in the case of Plato, and I should like to discuss it at some length. The term itself, or one of its positive cognates, occurs in twenty-eight

9. This interpretation was first proposed by Laloy 1900; cf. Barker 2007, 375–78. For other interpretations of the enigmatic "single name" thesis, see Barbera 1991, 55–58, and for a new hypothesis see Acerbi (forthcoming). One-word names for epimeric ratios were coined in later periods, perhaps first by Nicomachus (as suggested by Barbera 1991, 57).

10. In *Sect. can.* proposition 11, the writer argues on the basis of his principle that since the double fourth is a discord its ratio cannot be multiple. But the reasoning is plainly invalid.

distinct passages in the dialogues.[11] Although many of them associate συμμετρία with some sort of excellence or perfection, only three link it explicitly with τὸ καλόν; they are *Philebus* 64d–66b, *Timaeus* 87c–e, and *Republic* 530a. Plato makes no attempt to define or explain what he means by συμμετρία in any of these passages. In a well-known passage of the *Theaetetus* (147d–148b), however, where the mathematician Theaetetus is speaking, the concept is lucidly explicated in mathematical terms; it corresponds to what we call "commensurability," matching the definitions of συμμετρία given in Euclid's *Elements*.[12] *Parmenides* 140c certainly calls for the same interpretation, and so, perhaps, do the various passages in which a sense organ is said to be σύμμετρος with its objects (*Meno* 76d, *Tht.* 156d, *Ti.* 67c). Although none of these passages has anything to do with τὸ καλόν, we might be encouraged to try to give συμμετρία the same sense in those where the connection is made. But there are also reasons for being less than optimistic about the chances of this interpretation even before we have made the attempt, since it turns out to be entirely inappropriate in any of the passages I have not yet mentioned, that is, in twenty out of the original twenty-eight.

I shall not go through all the remaining twenty individually. Typical examples include *Timaeus* 89e–90a and *Laws* 918b. In the former, we are told that each of the three parts of the soul becomes stronger when it exercises its own special form of movement; hence, they will only remain σύμμετροι with one another if we regularly activate all of them. The gist of the latter is that when goods of any sort are distributed in a way that is uneven and ἀσύμμετρον, anyone who rectifies the distribution by making it even and σύμμετρον is a benefactor. It seems clear that what συμμετρία stands for in these passages is something like "due proportion." It would make no sense here to insist on the sense "commensurability"; and the same is true in almost all the other cases.

Let us now turn to the contexts in which συμμετρία and τὸ καλόν appear together. The theme of *Republic* 530a is that even the most beautiful (κάλλιστα) visible objects, such as paintings or sculptures, cannot be treated as sources of truth about "the equal or the double or any other συμμετρία." Perhaps this is intended to imply (though it is not expressly stated) that if a thing is σύμμετρον it is therefore καλόν. One might argue from the ex-

11. Or twenty-nine, if we include *Epinomis* 991b. A search in Brandwood 1976 or the online *TLG* will pick out well over thirty occurrences of relevant terms, but this is because they sometimes occur several times in the same passage. The negative forms ἀσυμμετρία, ἀσύμμετρος appear only three times in the corpus, only one of which (at *Grg.* 525a) is not in one of the twenty-eight passages I have mentioned. More commonly the word used to indicate absence of συμμετρία is ἀμετρία. Together with its cognates it appears twenty-four times in the dialogues (excluding *Epin.*, *Letters*, and *spuria*); none of these cases tells us anything more about the concept of συμμετρία than we can gather from the twenty-eight passages containing the positive term, and neither does the instance of ἀσυμμετρία in *Gorgias*.

12. *Elements* 10, Defs. 1 and 2. Def. 1 defines σύμμετρα μεγέθη; Def. 2 explains what is meant by saying that straight lines are δυνάμει σύμμετροι, which links directly with the issues that Theaetetus talks about in Plato's dialogue. In both cases, the concept defined is not proportion or balance or symmetry; it is commensurateness, or commensurability. Thus, the first definition, the only one directly relevant to our concerns, says that σύμμετρα μεγέθη are those measured by the same μέτρον, while ἀσύμμετρα μεγέθη are those of which there can be no common measure.

amples of the equal and the double that a συμμετρία here is a quantitative relation between whole numbers and that the terms of the relation must therefore be commensurable. This seems probable, though it cannot be guaranteed, since the notion of συμμετρία at work here might be capacious enough to include, for instance, the relation (in a suitable context) between the side and the diagonal of a square, a relation that qualifies as δυνάμει σύμμετρον in Euclid and the *Theaetetus*. But even if the terms must be whole numbers and must therefore be straightforwardly commensurable, this gives no grounds for insisting that anything whose constituents are commensurable with one another will therefore automatically qualify as σύμμετρον, still less that it must necessarily be καλόν. The passage may imply that the commensurability of a thing's parts is a necessary condition of its συμμετρία and beauty, but it certainly does not imply that it is sufficient. The συμμετρία that ensures its beauty must involve some particularly satisfying and appropriate pattern of relations, whether or not it is one of those that satisfy the condition of commensurability.

The intricate passage at *Timaeus* 87c–e deserves to be quoted in full, though I shall not examine all its details:

> Now everything ἀγαθόν is καλόν, and τὸ καλόν is not ἄμετρον; so it must be posited that a living creature too, if it is to be καλόν, must be σύμμετρον. We perceive and calculate [συλλογιζόμεθα] the small instances of συμμετρίαι, but we are unreasoning in the face of the most important and greatest of them.[13] For in connection with health and diseases, and virtues and vices, there is no greater συμμετρία and ἀμετρία than that of the soul itself to the body itself; but we investigate none of these, nor do we notice that when a soul that is strong and great in every way rides on a bodily form that is weaker and smaller, or when the two are put together in the opposite way, then the whole creature is not καλόν, since it is ἀσύμμετρον in the greatest συμμετρίαι, whereas for anyone who has the power to see, a creature in the opposite condition is the κάλλιστον and most lovable of all sights. By way of analogy, a body that is too long-legged or ἄμετρον with itself through some other excess is not only αἰσχρόν, but at the same time is . . .

This is the most elaborate of the Platonic passages in which συμμετρία and τὸ καλόν are explicitly linked, but for present purposes we can limit ourselves to two straightforward points. First, we might initially suppose that Timaeus is treating συμμετρία as only a necessary condition of beauty. But in fact his thesis is stronger than that. A creature that is "ἀσύμμετρον in the greatest συμμετρίαι" is not καλόν, but "a creature in the opposite condition is the κάλλιστον . . . of all sights." This plainly implies that συμμετρία, at least in this case, is an absolute guarantee of beauty. Secondly, it is even clearer here than in the previous passage that the notion of συμμετρία cannot be exhausted by that of commensurability, even if we concede that it requires it; the closing analogy shows that by itself. Suppose, for instance, that a person's legs will be σύμμετρα with their arms if the distance from the hip to the heel is in the ratio 2:1 to the distance between the elbow and the tip of the index

13. "We are unreasoning" translates ἀλογίστως ἔχομεν, perhaps "we fail to calculate." Alternatively, the meaning may be "but we grasp the most important and greatest of them nonrationally."

finger. They will be just as commensurable with the arms if the ratio is 4:1 or 7:1, but we can hardly suppose that Plato's Timaeus would have attributed συμμετρία and beauty to a body so bizarrely proportioned.

The question at issue in *Philebus* 64d–65a is about the identity of the factor primarily responsible for the excellence of any mixture. Socrates and Protarchus agree that it must be "μετριότης and συμμετρία," since in their absence the compound will be merely an unblended jumble, no longer a genuine mixture; and Socrates comments that the attribute "good" has now hidden itself in the attribute "beautiful," since, he says, "κάλλος and ἀρετή always turn out to be μετριότης and συμμετρία." We are unfortunately given no help with the interpretation of the key terms; it is not even clear whether μετριότης and συμμετρία are distinct items, or whether the phrase is a hendiadys, "μετριότης, that is, συμμετρία." The statements about mixtures are worth noting in passing, since they connect smoothly with the ideas we have met about the fusion of notes in a concord. Otherwise, the most we could infer from the passage, for present purposes, is that beauty and *symmetria* are again linked very closely—perhaps even identified with one another, if κάλλος and ἀρετή are coextensive and "μετριότης and συμμετρία" is a hendiadys. We can confidently assume that the συμμετρία of a mixture depends on the proportions of its constituents; but the conditions that these proportions must satisfy remain obscure.

There seems, then, to be a disjunction between two groups of passages connected with συμμετρία in Plato. In one group, it is to be identified with commensurability, certainly in the *Theaetetus* and the *Parmenides*, and probably in the others I mentioned in connection with them. In the other, much larger group, commensurability is never sufficient for συμμετρία, though in some cases it may be necessary for it; the governing idea seems to be that of "appropriate proportion," but what makes a proportion appropriate is never specified. This group contains all those in which συμμετρία is connected with beauty, and in at least two of the three cases, perhaps in all of them, the relation is very close. It would not be unreasonable to conclude from these passages, as perhaps Plotinus did, that Plato was in effect defining τὸ καλόν as τὸ σύμμετρον. But in the absence of any clear analysis of συμμετρία, the definition leaves a good deal to be desired.

Most of the mathematical theorists are dismissive about the evidence of sense perception; they ignore questions of the kind I have raised, and they seldom do much to ensure and to demonstrate that their mathematically constructed scales and attunements correspond to those actually used in practice by musicians.[14] Ptolemy is by far the most important exception.[15] He insists

14. See, for instance, the comments of Ptolemaïs of Cyrene and Didymus ὁ μουσικός on the theorists they call "Pythagoreans," quoted at Porph. *Harm*. 23.24–31, 25.9–14, 26.15–25 Düring. In many cases, such theorists' lack of attention to the empirical data is understandable, since they are more concerned with the role of harmonic structure in the cosmos at large and in the soul than with audible music as such.

15. Of the other exceptions the most interesting is Archytas; for an illuminating discussion of his treatment of the relation between mathematical reasoning and the evidence of the ear, see Huffman 2005, 410–25. It seems likely that Didymus and perhaps Eratosthenes were also trying to accommodate the systems of musical practice in their mathematical representations, but we can only speculate about the way they understood the relations between them. Ptolemy records both these theorists' harmonic divisions, along with those of

that the task of harmonics is to demonstrate complete agreement between the results of the mind's mathematical reasoning and the evidence of sense-perception. Reasoning, based on principles derived by abstraction from perceptual experience, must come first; but its conclusions cannot be accepted until they have been submitted to the judgments of the ear.[16] Correspondingly, the central purpose of his discussion of the concords and of musical beauty in general, to which we shall now turn, is to show how the ear and the mathematical mind are attuned, as it were, to the very same features and excellences of musical structure.

The musical intervals whose ratios need to be given mathematical credentials fall into two groups, concords on the one hand, and simple scalar intervals, the ἐμμελεῖς or "melodic" intervals, on the other. The term συμμετρία and its cognates enter explicitly only into Ptolemy's account of the ἐμμελεῖς intervals, but we shall find that the considerations underlying their use in that context apply with equal force in the other. The essential condition that Ptolemy lays down in his *Harmonics* for all "melodic" ratios is that their terms should be ἐν συμμέτροις ὑπεροχαῖς, literally "in commensurate excesses." We meet the expression in connection with the ratios of individual scalar intervals in *Harmonics* I.7, for instance (16.13 Düring); and in I.13 he commends Archytas for having "tried to preserve what is in accordance with reason not only in the concords but also in his divisions of the tetrachords, on the grounds that the commensurateness of the excesses [τὸ σύμμετρον τῶν ὑπεροχῶν] is proper to the nature of melodic intervals" (30.9–13 Düring).

The meaning of these rather awkward phrases is best brought out in a passage of I.5, which pivots on another of Ptolemy's favorite phrases, ἁπλότης τῆς παραβολῆς, "simplicity of comparison." Here he is offering a diagnosis—probably his own, not one he found stated explicitly in earlier texts—of the reasons why the theorists he calls "Pythagoreans" gave privileged status to multiple and epimoric ratios;[17] and he makes his approval clear, though he adds some qualifications later (*Harm.* 11.8–17 Düring):

> They laid down a first principle for their method, which was entirely appropriate, according to which equal numbers should be associated with equal-toned notes, and unequal numbers with unequal-toned; and from this they argue that just as there are two primary classes of unequal-toned notes, the concords and the discords, and that of the concords is finer [or "more beautiful," κάλλιον], so there are also two primary distinct classes of ratio between unequal numbers, one being that of what are called "epimeric" or "number to number" ratios, the other being that of the epimorics and multiples; and of these the

Archytas and Aristoxenus, in the tables set out in *Harm.* II.14. He comments critically on those of Didymus in the preceding chapter, and mentions certain innovations in his use of the monochord, apparently designed to allow the credentials of his divisions to be made more readily apparent to the ear. About Eratosthenes' divisions he says nothing at all.

16. Ptolemy first announces his conception of the harmonic theorist's task at *Harm.* 5.13–24 Düring, going on to denounce the majority of his predecessors for having neglected one or other of the two criteria (5.24–26.13). The ways in which his methodology in the treatise as a whole is guided by this conception are discussed in Barker 2000. For translations of *Harm.* with substantial commentaries, see Barker 1989, 270–391 (though it contains a number of uncorrected errors); Solomon 2000; Raffa 2002.

17. This reflects the thesis of *Sect. can.* discussed above. Ptolemy certainly knew the treatise, and drew directly on it later in this passage (12.8–27, which paraphrases five of *Sect. can.*'s propositions).

> latter is better than the former κατὰ τὴν ἁπλότητα τῆς παραβολῆς ["on account of the simplicity of the comparison"], since in the case of the epimorics the excess is a simple part, while in the multiples the smaller term is a simple part of the greater.

The better ratios, then, are those that make possible a straightforward comparison between the sizes of the terms. In multiple ratios, the smaller term is a "simple part" of the larger, and the comparison consists in saying how many times greater the larger term is; the smaller is a unit by which the larger can be measured. In epimoric ratios, the "measure" is the difference between the terms, the "excess," since it is always a "simple part" of both of them (this is obvious from the fact that when such ratios are expressed in their lowest terms the difference between them is always 1). The essential point is that the ratios should include within themselves an element that constitutes an appropriate "measure"; and this is not the case with the epimerics.[18] No component of the ratio 7:5, for instance, will serve as the unit in relation to which the two terms can be compared.

When Ptolemy insists that the ratios of the melodic intervals must have "commensurate excesses," he is, in fact, telling us that they must have the feature that he ascribes in this passage to the epimorics; the "excess," that is, the amount by which the greater term exceeds the smaller, must be commensurable with both terms, so as to function as the unit that mediates comparisons between them. In short, all melodic intervals must have epimoric ratios. We can easily see why he does not speak of "commensurate excesses" when he is focusing on the ratios of the concords, using instead the more inclusive notion of "simplicity of comparison"; it is because some ratios of concords, 2:1 (the octave), 3:1 (the octave plus a fifth), and 4:1 (the double octave), are not epimoric but multiple, and in their case the difference between the terms need not be commensurate with them. But the comparison remains "simple," since the smaller term itself is commensurate with the greater and is the common unit of measurement. Thus "simplicity of comparison" can be realized in either of two ways, only one of which requires that the terms must be ἐν συμμέτροις ὑπεροχαῖς; but both of them involve commensurability, συμμετρία.

This is not quite the end of the matter. The questions about correctness of form, in a harmonic context, are not restricted to ones about the ratios of individual intervals, whether concords or melodics. They deal also with the structures of systems of ratios; and, in fact, the questions about ratios of intervals themselves cannot be settled without consideration of the structures in which they are embedded, since the ratios are worked out by a process of dividing some large interval, typically the octave, into its smaller scalar components. Ptolemy's first step is to divide the ratio of the octave, 2:1, into the two epimoric ratios that most nearly divide it in half, those of the fourth and the fifth, 3:2 and 4:3. The next step presupposes, as is normal in Greek

18. This is a point that was grasped perfectly by Porphyry in his commentary; he expresses it lucidly at *Harm*. 98.5–13 Düring. (I refer to Düring 1932 in citations of Porphyry and to Düring 1930 in page and line citations of Ptolemy.)

musical theory, that the main landmarks in any straightforward octave scale lie a fourth from the bottom and a fourth from the top, separated by a tone in the ratio 9:8, and that the two fourths are divided in exactly the same way, to form two identical tetrachords. What is needed, then, in order to produce a complete catalogue of the legitimate forms that an octave scale can take, is an exhaustive analysis of the acceptable ways in which the ratio of a fourth can be divided.

Ptolemy reaches this goal in two stages. He first divides the ratio of the fourth into two epimoric subratios, as many times as is mathematically possible; and secondly, he takes, in turn, one or other of the two subratios in each division and divides it also into two epimorics, leaving the other subratio untouched (see I.15). Then, since every division along the way divides a ratio into two epimorics, the term inserted between those of the original ratio will always be σύμμετρος, "commensurable," in Ptolemy's sense of the word, with both the terms of the ratio it divides. Thus, the whole system of notes in any acceptable scale is held together by a network of συμμετρίαι, brought to light by the process of division through which Ptolemy constructs them.

These, then, are the ways in which συμμετρία enters into Ptolemy's theory, very pervasively and in a thoroughly Euclidean guise. Let us grant that his reasoning makes mathematical sense. But we obviously need to ask how it relates to our aesthetic perception of musical relations. Why should we suppose that intervals that strike the ear as musically beautiful concords, or as acceptable steps of a legitimate musical scale, must correspond to ratios with features of this particular sort, or that a whole scale must always be woven together in this manner? In what follows I shall consider only the simpler part of this question: how can Ptolemy justify his evident belief that an interval's possession of such a ratio is a necessary condition of its being an element in an aesthetically pleasing musical system?

To answer that question we have to go back to the first chapter of the treatise, where Ptolemy offers a general discussion of the relations between sense perception and reason. Let us consider first a rather breathless and convoluted passage which—once we have untangled it—will put us on the track of the governing idea (*Harm*. I.1 [3.3–14 Düring]):

> The criteria of *harmonia* are hearing and reason, but not in the same way. Rather, hearing is concerned with the matter and the πάθος,[19] reason with the form and the cause, since it is in general characteristic of the senses to discover what is approximate and to adopt from elsewhere what is accurate, and of reason to adopt from elsewhere what is approximate and to discover what is accurate. For since matter is determined and bounded only by form, and πάθη only by the causes of movements, and since of these the former [matter and πάθος] fall into the province of sense-perception, the latter [form and cause] into that of reason, it follows naturally that the apprehensions of the senses are determined and bounded by those of reason, first submitting to them the distinctions they have grasped in rough outline—at least in the case of the things that can be detected through

19. I take the word πάθος, as Ptolemy uses it here and elsewhere, to refer to the perceptible attribute that the "matter" acquires through the agency of the "cause" mentioned in the next phrase.

> sensation—and then being guided by them toward distinctions that are accurate and agreed.

Some aspects of the thesis put forward here are depressingly familiar. Perception is rough and ready and unreliable; reason is an accurate and unwaveringly consistent arbiter of the truth. Ptolemy rehearses these ancient saws in his next sentence, adding some desultory metaphysical underpinnings, which need not concern us. But it is clear from the things I have said already that he cannot possibly rest content with these philosophical commonplaces. When reason has established its principles and derived its mathematical constructions, its conclusions must be submitted to the judgment of the ear; perception, as he says in the first sentence I quoted, is one of the "criteria" of *harmonia*. It would be a poor candidate for such a role if it had no better qualifications than the ones we have so far extracted.

But the passage conveys a more positive point. The senses, we are told, not only "discover what is approximate" but also "adopt from elsewhere what is accurate." The same idea reappears in the clause with which the passage ends: the senses are guided by the apprehensions of reason "toward distinctions that are accurate and agreed." Ptolemy seems to mean that once reason has done its work on the rough impressions conveyed by the senses, the senses will recognize and accept the authority of reason's judgments, and their impressions will thereby be corrected and made accurate.

The next part of the passage also helps us to identify the grounds on which Ptolemy bases these contentions; he offers an example to clarify what he has in mind when he says that the senses will be guided by reason towards accuracy (*Harm*. I.1 [3.20–4.7 Düring]):

> Just as a circle constructed by eye alone often appears to be accurate, until the circle formed by means of reason brings the eye to a recognition of the one that is really accurate, so if some specified difference between sounds is constructed by hearing alone, it will commonly seem at first to be neither more nor less than what is proper; but when there is tuned against it the one that is constructed according to its proper ratio, it will often be proved not to be so, when the hearing, through the comparison, recognizes the more accurate as legitimate, as it were, beside the bastardy of the other.

As we discover at the end of the chapter, the circle "formed by reason" is one drawn in such a way that it fits the mathematical definition of a circle, that is, one constructed with the aid of the compasses, a device designed specifically for this purpose. When we are dealing with musical intervals we shall need acoustic counterparts of the compasses, gadgets that can display specified intervals to our ears in the form that mathematical reason assigns to them; these are the monochord and the various more complicated instruments that Ptolemy describes later in the *Harmonics*. Reason discovers exactly what must be done to a line if it is to describe a circle and what must be done to a pair of sounds if they are to stand in the relation of a perfect fourth, and the practical devices allow us to lay out the rationally constructed figure or interval in front of our eyes or our ears. Then, Ptolemy asserts, our senses will unfailingly recognize the superiority of the rationally constructed example over the imperfect specimen that had previously satisfied it.

The example of the circle is persuasive, and if we take, for instance, the construction and identification of a perfect fourth as an example in the acoustic sphere, it has at least enough plausibility to pass muster. We should note that the assumption that perception will accept reason's authority in such cases does not undermine Ptolemy's proposal that reason's findings should be tested against the judgments of the ear. This implies that perception may sometimes reject the conclusions that reasoning purports to have established, rather than automatically deferring to them; but the point is that human reasoning can, after all, make mistakes, and the senses have sufficient independence to challenge its findings if they conflict with their impressions. Ptolemy's contentions in the present passage can, in fact, be used to support his proposals about empirical testing. If perception will invariably accept the constructions of reason when they are correct, the fact that in certain cases it refuses to accept them is a reliable sign that the reasoning has gone astray.

But I shall not dwell on that issue now. We are still waiting to find out why the intervals perceived by our ears as musically well formed should abide by the mathematical recipe that Ptolemy gives, that is, that their ratios should conform to the "simplicity of comparison" criterion. Unless that question can be answered, there will be no grounds for supposing that reason's constructions have any bearing on the nature of the systems that musicians and their audiences find aesthetically admirable, or for the expectation that our musical perceptions will obediently accept the correction offered by the mathematicians, no matter how cogent their logic. The mathematicians and their constructions will be left roaming the soundless metaphysical wastelands, in company with Pythagoreans and Platonists.

Ptolemy's answer to the question seems to be contained in the next part of his first chapter, which continues his discussion of the senses' capacity to reach an accurate assessment of the phenomena with which they engage. In particular, he is thinking of what we do when we compare two different items, in the sense that we are trying to identify the relation between them. Perception, he says, can quite easily tell when two things are different, for instance, when one is bigger than the other, and it is fairly reliable when it comes to judging "the amounts by which differing things exceed one another, so long as the amounts in question consist in larger parts of the things to which they belong" (4.10–13 Düring). As shortly becomes clear, he is not thinking of the "amount" by which one thing exceeds the other as an absolute measurement, so many feet or inches, for example, but in terms of its relation to the other item; the greater length exceeds the smaller by some "part" of the smaller. Judgment of the relation is not too hard when the part in question is large in relation to that of which it is a part, half of it or one third of it, for example. But it becomes progressively more difficult, he continues, as the part becomes a smaller fraction of the whole; it is much harder, for instance, to judge accurately by eye alone when one thing is longer than another by one seventh.

Ptolemy's contentions here are unlikely to provoke disagreement, and we need not pursue the way in which he accounts for the difficulty of assessing the smaller fractions. Let us try instead to apply his statements in the context

of the ear's attempts at judgment, for instance, when we are trying to decide, by ear, whether the relation between two notes that we hear is or is not a perfect fourth. To make them fit at all closely, we shall have to make at least one crucial assumption that might well be challenged. It is that the relevant difference between the two notes we hear is of a quantitative sort and, more precisely, that it can be specified by identifying the fraction of some quantity attached to one of the notes by which the other note's corresponding quantity exceeds it. Since the attributes of the notes with which we are concerned are their pitches, and since the faculty by which we are comparing them is our hearing, the assumption is that when two different pitches are presented to our ears the difference between them is a matter of relative quantity.

Ptolemy defends this assumption in I.3. I said that it is open to challenge, and though Porphyry in his commentary on the *Harmonics* rarely dissents from the views expressed in the text he is discussing, he attacks both the assumption itself and Ptolemy's defense of it with remarkable vigor and cogency, and at considerable length.[20] I shall not examine the arguments here; our business is with Ptolemy, so let us register his assumption and move on. The fact that in I.1 it is still no more than an assumption may explain why he deploys only examples of visual judgments there and not musical ones; he cannot rely on his readers to think of pitch difference in his quantitative manner until he has demonstrated, to his satisfaction, that it is correct. The same consideration may also explain why he only implies, and does not explicitly state, that if a satisfactory comparison is to be made, the unit by reference to which the two quantities are compared must constitute either the whole quantity attached to the smaller item or a "simple part" of it, one third or one seventh or the like. The classification of ratios that he attributes to the Pythagoreans, along with his association of the epimorics and multiples with "simplicity of comparison," is reserved for a later stage.

Ptolemy's conception of the connection between aesthetically satisfying musical relations and numerical ratios whose elements include a unit by which both terms can be measured should by now be clear, at least in outline. In forming our perceptual judgments, we are performing a kind of subconscious mental arithmetic, comparing one quantity with another. We do not need to be aware, of course, that we are doing anything of the sort, any more than we need to be thinking in geometrical terms when a well-proportioned piece of architecture strikes us as elegant. Further, the mathematical character of the relation determines the degree of aesthetic enjoyment, because the easier the process of comparison is, the more pleasing we find the relation. This is a point that Ptolemy brings out piecemeal in I.5 and I.7. In I.5, he tells us first that concords, as a class, are "finer" or "more beautiful," κάλλιον, than discords; they should therefore be correlated with multiple and epimoric ratios, which are "better," ἄμεινον, than epimerics "because of the simplicity

20. The main phase of his critique is at 55.30–61.15 Düring, but he has been preparing for it through much of his discussion of earlier parts of Ptolemy's chapter (which begins at 29.27), and goes on to quote extensively from two other writers (down to 66.15) in support of his anti-Ptolemaic stance.

of the comparison," as we have already learned. Shortly afterwards he offers reasons for saying that the octave is the most beautiful, καλλίστη, of the concords, and must therefore be linked with the "best," ἄριστος, of the ratios, which is 2:1 (11.10–24 Düring). In I.7 he explains that the intervals next after the concords in excellence, ἀρετή, are the melodics, so that their ratios must be the epimorics that come after the ratio of the fourth, 4:3, following the sequence 5:4, 6:5 and so on, and that "those that make divisions most nearly into halves must be more melodic, . . . as are all those whose differences contain larger simple parts of those that are exceeded" (16.12–21 Düring). All the ratios involved must have terms in a relation that conforms to the general criterion of "simplicity of comparison," but the corresponding intervals become progressively less beautiful as the ratios become "worse" and the comparison becomes more difficult. It is more difficult, for instance, when we are comparing pitches in the ratio 6:5, where the common measure is one fifth of the smaller term, than it is when they are in the ratio 3:2, where half of the smaller term serves as the common measure. An interval in the ratio 6:5, a kind of minor third, is therefore less beautiful than the concord of a perfect fifth.

Ptolemy's explanation of the fact, as he takes it to be, that the judgments of reason and hearing chime together is, thus, in essence, that although they present their verdicts in different forms, they are, in fact, comparing the same quantitative data, roughly or accurately, in exactly the same way. Suppose, then, that we hear two notes whose frequencies, to put it in modern terms, are not in fact related as 4:3, but for instance as 4:2.9; perhaps one has a frequency of 400 Hz and the other of 290. Most of us would probably identify the interval as a musically satisfactory fourth. But if we were then asked to compare it with one constructed in the correct ratio, as Ptolemy suggests, it is indeed likely that we would recognize the latter's superiority; Ptolemy seems to be quite right about this. The point is that if the tuning of two strings, for instance, is gradually adjusted towards the relations of a fourth, a fifth, or an octave, there comes a moment when the interference between the two pitches reaches a minimum and the combined sound becomes maximally smooth; and it is at precisely the point at which the ratio is in Ptolemy's sense the "simplest." One can go on in the same way a little beyond this; the relation that we recognize as the concord of a major third is at its clearest and sweetest when the ratio is exactly 5:4. But as one moves from here to still smaller intervals, when the terms of the ratios get bigger and the difference between them gets relatively smaller, it becomes progressively harder to decide when the "correct" relation has been reached. And this, too, corresponds precisely to what Ptolemy has said about the increased difficulty of making comparisons when the two items differ by only a small fraction of the smaller term.

One might argue that the notion of the "correct" relation has no real application when we are dealing with small intervals. Certainly, it seems implausible to say that one version of a halftone is perceptibly sweeter or more beautiful than another. If we have reasons for preferring one version in particular, it will probably have more to do with its relation to other intervals

appearing in the same context and its role in the musical flow of the passage than with the way it sounds in isolation. Ptolemy, I think, would agree, at least up to a point; he has mathematical grounds for identifying the ratio of an interval approximating to a halftone as 16:15 in some contexts, as 15:14 in others, and as different again in yet other legitimate systems. But he continues to insist on the thesis that all such ratios must be epimoric, in accordance with his principle of "simplicity of comparison" and συμμετρία, even though there can hardly be compelling auditory evidence that it still holds good in these difficult cases (as, at one point, he implicitly admits; see 39.14–40.8 Düring).

I want to end by saying something about remarks that Ptolemy makes much later in the treatise, in III.3, when the main business of the *Harmonics* is already complete. Here he leaves technical matters behind him and reflects in more general terms about the nature and status of the concepts and principles he has been using. It is a philosophically fascinating passage, which deserves careful study. But all I can do here is to sketch very briefly the gist of the first part of the chapter, and then draw attention to the ways in which he goes on to outline the relations between reason and hearing, and between both of them and beauty, τὸ καλόν.

After a rather grandiose introductory paragraph (91.22–93.8 Düring), Ptolemy begins by reviewing a tripartite version of the Aristotelian catalogue of the "four causes," represented as matter, moving cause or agency, and form combined with τέλος. He draws the conclusion that *harmonia* falls into the category of agency. Next, he divides such agencies, "at the highest level," into three kinds: one is φύσις, which is responsible only for bringing things into being; the second is reason, λόγος, which brings nothing into being but is responsible for τὸ εὖ εἶναι, that is, for things "being good"; and the third is god or the divine, which is responsible for τὸ εὖ καὶ ἀεὶ εἶναι, "good and eternal being." *Harmonia*, he concludes, is a form of λόγος, which cooperates with the two agencies that cause things to exist by making their products good (92.9–24).

Now we get another triple division (92.24–26); dividing things into three parts or types is a favorite Ptolemaic strategy. One aspect of λόγος, conceived as agency, is νοῦς, intellect, whose task is to apprehend the appropriate form. The second is τέχνη, practical art or skill, by which things are molded into the pattern that νοῦς has discovered. The third is ἔθος, roughly "habituation," through which, for example, a disposition in accordance with τὸ εὖ εἶναι becomes assimilated into a human soul. (That, at least, is how I understand Ptolemy's remarks about this third aspect of λόγος; he alludes to it three times in different formulations, but each is as opaque as the others.)

λόγος as such, Ptolemy says, is responsible for τάξις and συμμετρία. The special variety of it that is concerned with what is heard, ἁρμονικὸς λόγος, "puts right the τάξις in audible things, to which we give the specific name ἐμμέλεια, through the theoretical discovery of συμμετρίαι by means of νοῦς, through their exhibition in products of manual labor by means of τέχνη and through experience in following them by means of ἔθος." Correspondingly, he adds, in an intriguing extension of this thesis, "the science that embraces

all the forms of λόγος, which we call μαθηματική, is not concerned solely with the theoretical grasp of beautiful things, τῶν καλῶν, as some people suppose, but also with their exhibition and cultivation, which arise from the act of pursuing them" (92.27–93.10). Such an inclusive and engaging conception of μαθηματική is surely to be commended to modern mathematicians. For present purposes, however, what we should note in addition is the smoothness with which Ptolemy moves between references to συμμετρίαι and to τὰ καλά; they are two sides of the same coin.

It is obvious that reason cannot do all this work by itself; but help is at hand. "This faculty," Ptolemy continues, "uses as it were as its instruments and servants the highest and most marvelous of the senses, sight and hearing, which of all the senses are the most closely allied to the ruling principle" (93.11–13). We may well wonder what warrant he has for saying this; but he immediately explains. It is that they are the only senses "that judge their objects not only by the criterion of pleasure, but also, much more importantly, by that of beauty, τὸ καλόν." All the senses discriminate attributes within their own domains, he goes on, and all of them distinguish between agreeable and unpleasant qualities; "[b]ut no one would locate τὸ καλὸν ἢ αἰσχρόν among the objects of touch or taste or smell, only among those of sight and hearing" (93.14–22).

Ptolemy's position could hardly be clearer. The faculty that gives us authoritative access to the nature of τὸ καλόν is mathematical reason, specifically "harmonic reason," ἁρμονικὸς λόγος, when we are concerned with beauty in the audible domain. Since τὸ καλόν consists in forms of συμμετρία, and since our hearing, too, has the capacity to distinguish between instances of τὸ καλόν and τὸ αἰσχρόν, it must in these cases be recognizing, at least implicitly, when συμμετρία is present and when it is not. It can, therefore, both provide mathematical reason with the rough data it needs to work on and cooperate with it in the phases of its activity that involve τέχνη and ἔθος; and as we have seen, it can also legitimately register its protests if reasoning goes astray, as happens all too frequently in the cogitations of mere mortals.

There is much more of interest in this reflective chapter, including a rather charming development of an image pioneered by Archytas and Plato (DK 47B1; *Resp*. 530d), which admirably conveys the essence of Ptolemy's position. The acolytes of reason, the beauty-detecting senses of sight and hearing, are sisters, and the "most rational" sciences that depend on them, astronomy and harmonics, are these sisters' daughters and one another's cousins, nourished and educated under the tutelage of geometry and arithmetic (94.9–20). The final upshot of Ptolemy's work is that reason is no longer left in dignified isolation, contemplating the eternal beauties of the divine *harmonia* but exercising no control over the wanton warblings that enchant the ears of audiences in the theaters of Athens and Alexandria. Through maneuvers conducted in I.16 and more intricately in II.1 and II.16, he explains in detail how his theoretical constructions are related to the tuning systems used by the musicians at work in his own environment, and provides his readers with all the instructions they need in order to exercise their hearing and their aesthetic sensibilities in judging for themselves whether

or not his conclusions are correct. He is confident that they will agree that they are.

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RESPONSE TO BARKER

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Andrew Barker provides a lucid exposition of Ptolemy's attempt to explain what accounts for the beauty of the music we hear, the beauty of the mathematical relations that govern what we hear, and the connection between the two. His excellent work is, however, problematic for me as a commentator, since I find myself in virtually complete agreement with his account of Ptolemy's position. My comments will thus support and supplement what Barker says rather than contradicting it. In what follows, I will first examine two further examples, Polyclitus and the Pythagoreans, that largely support Barker's account of the role of the central concept, *symmetria*, in Greeks' accounts of beauty prior to Ptolemy. Then, I will argue that the Greek harmonic tradition, including Ptolemy himself, misrepresents the historical development of the antecedents of Ptolemy's theory of beauty among Plato and the Pythagoreans. Although, as a whole, Ptolemy goes far beyond the Pythagoreans in his explanation of beauty, some parts of his account, in fact, represent a return to the Pythagorean position.

As Barker has shown, it was a very common Greek instinct to try to define beauty in terms of *symmetria*, which he translates as "due proportion" or "balance." Thus, Plotinus asserts that beauty "is said by practically everyone to be *symmetria* of parts to one another and to the whole" (*Enn*. 1.6).[1] Barker has also drawn our attention to what appears to be a disappointing failure of most Greek thinkers to say in any precise way what *symmetria* is; ratios are involved, perhaps, but no one steps forward to say which ratios, let alone explain why it is that these particular ratios, rather than some others, produce beauty. It is instructive to examine another example of this Greek fascination with and elusiveness about *symmetria*, the fifth-century Argive sculptor Polyclitus, whom Barker mentions in passing. In a famous fragment from his book *The Kanôn*, he says that "the good" (τὸ εὖ), which in context must mean a good and hence beautiful sculpture, "arises just barely through many numbers" (παρὰ μικρὸν διὰ πολλῶν ἀριθμῶν γίνεται).[2] Although Polyclitus does not use the word *symmetria* here, there can be no doubt that the numbers involved were the numbers in the ratios of the size of the various parts of the body to one another and the whole, and that he was thus defining

1. All translations are my own unless otherwise indicated.
2. DK 40B2.